Multibody drivetrain model used for predicting gear tooth stresses in a planetary gearbox

Jørgensen, Martin Felix

Publication date:
2013

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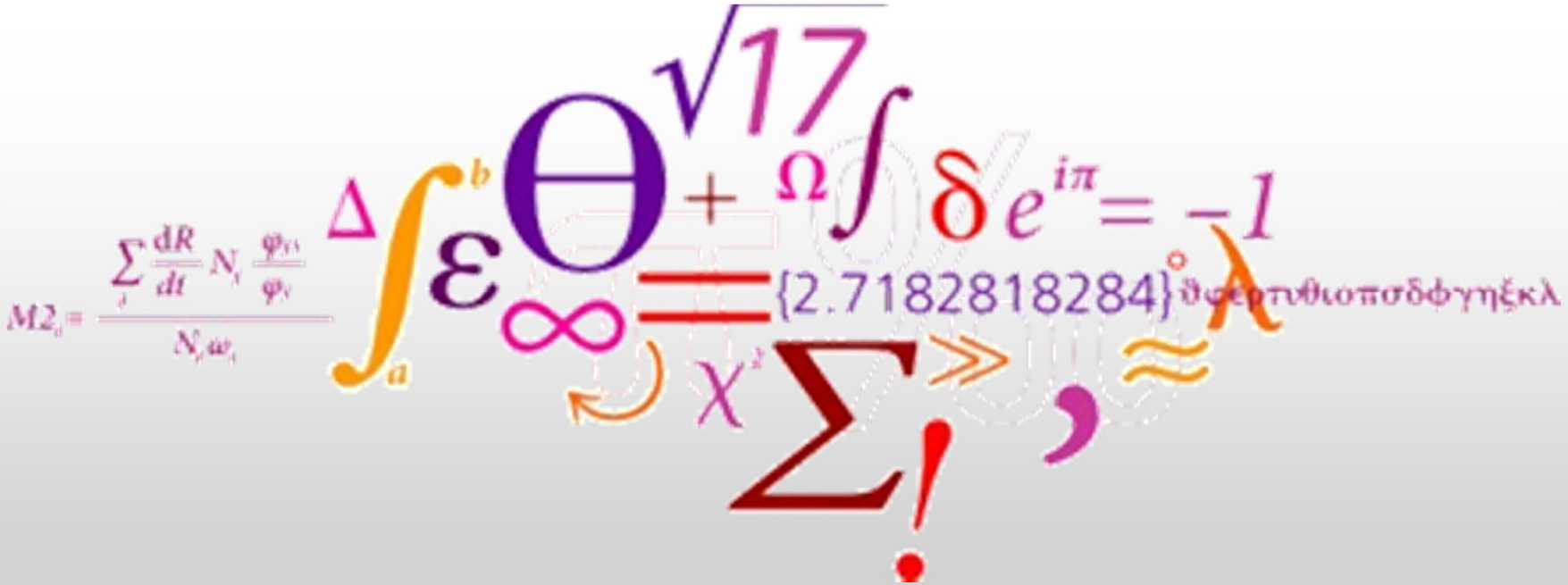
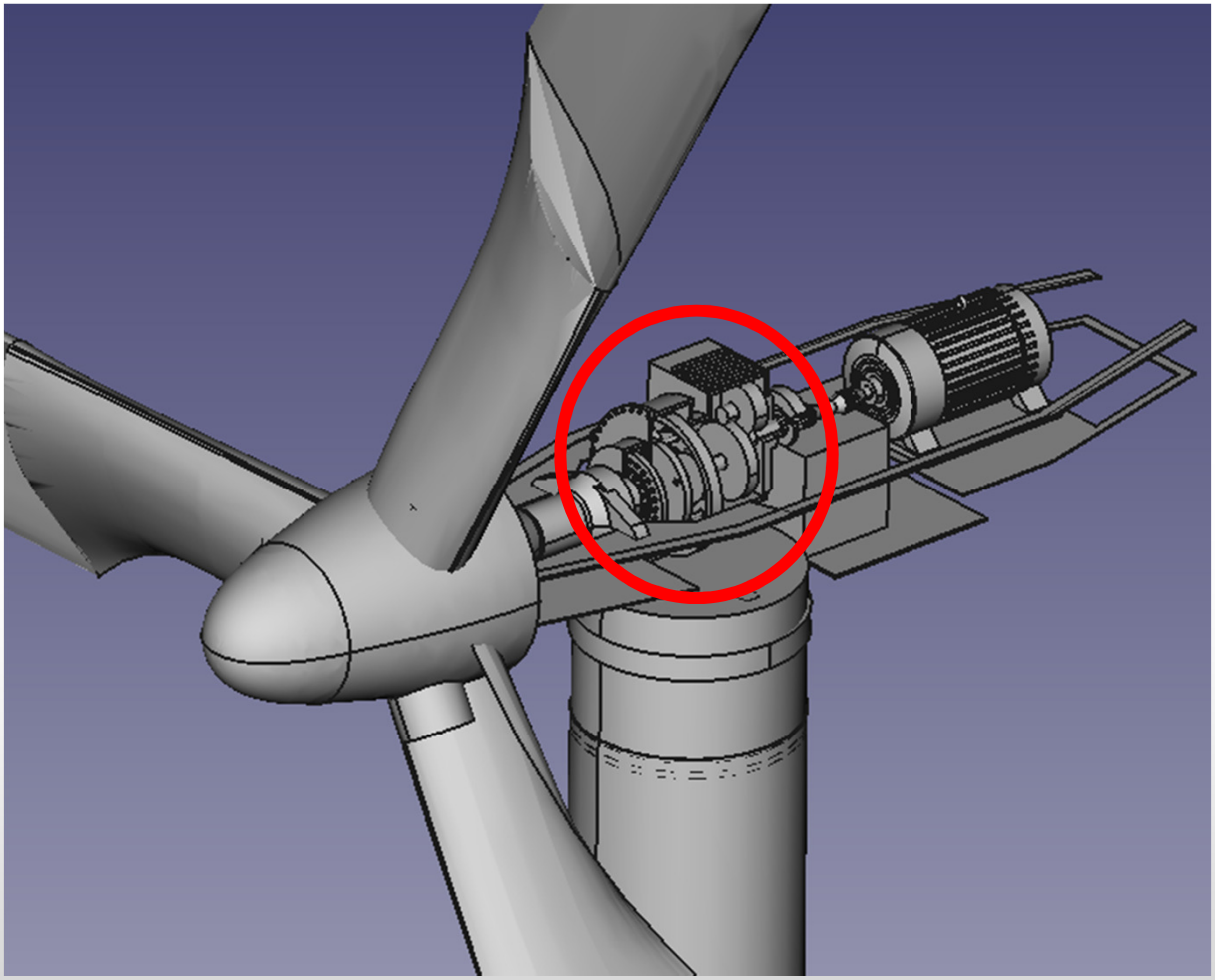
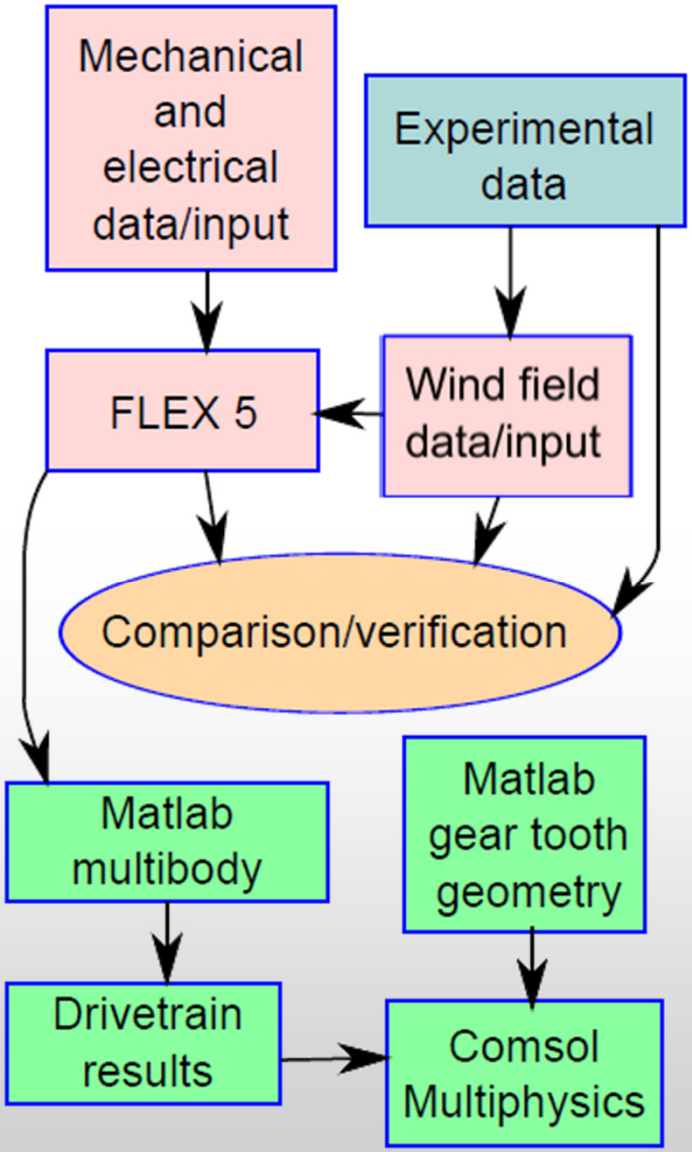
Martin F. Jørgensen, PhD student



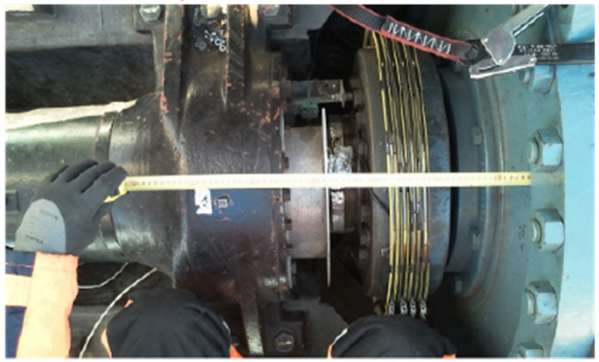
DTU Mechanical Engineering

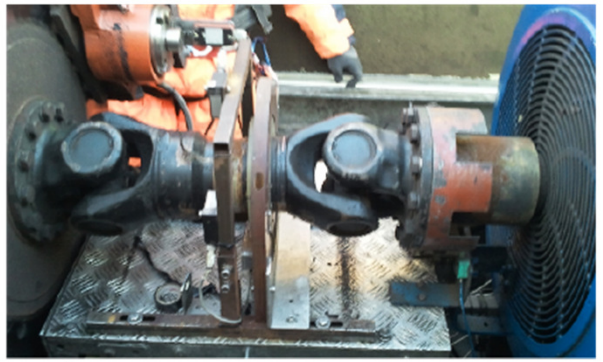
1. Introduction

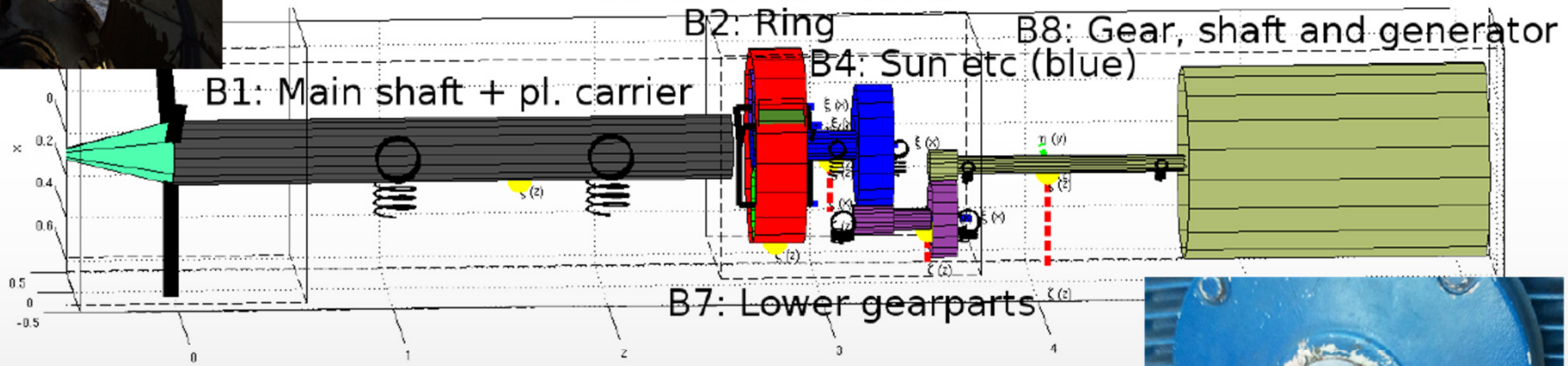




2. Multibody Model







3. Equations of motions (constraints)

$$\begin{bmatrix} M & \Phi_q^T \\ \Phi_q & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ -\lambda \end{bmatrix} = \begin{bmatrix} g \\ \gamma \end{bmatrix}$$

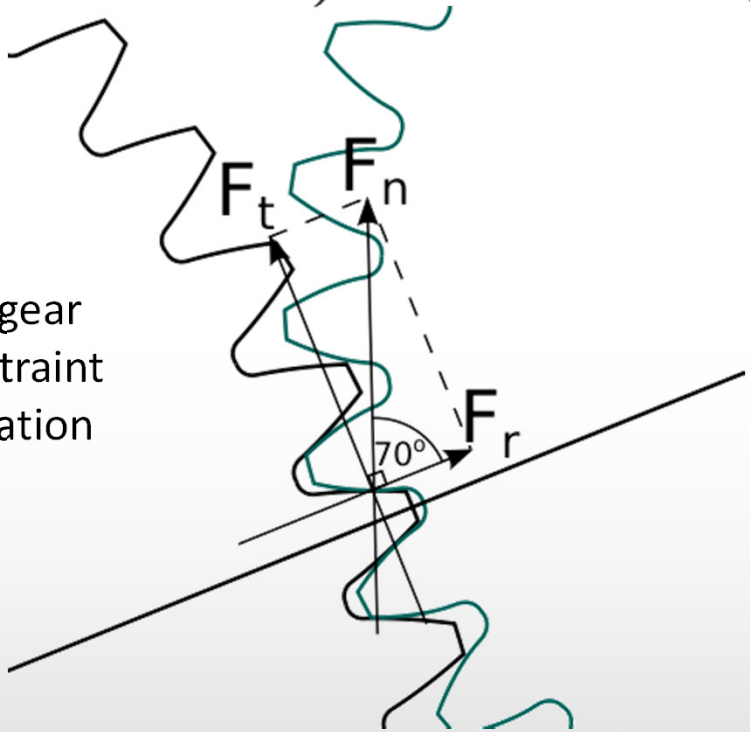
Reaction forces (in bearings/gear tooth forces etc):

$$M\ddot{q} = \sum F \quad \text{or:} \quad M\ddot{q} = \sum F_{ext} + \sum F_{react} \Rightarrow M\ddot{q} - \underbrace{\Phi_q^T \lambda}_{\text{reaction forces}} = F_{ext}$$

4. Rigid gear constraint (vs. flexible)

$$\dot{\Phi} : (A_{70} \cdot v_r)^T \left(\begin{Bmatrix} \dot{x}_1 \\ \dot{y}_1 \end{Bmatrix} + r_{p1}(\omega_1 \hat{v}_r) \right) - (A_{70} \cdot v_r)^T \left(\begin{Bmatrix} \dot{x}_2 \\ \dot{y}_2 \end{Bmatrix} - r_{p2}(\omega_2 \hat{v}_r) \right) = 0$$

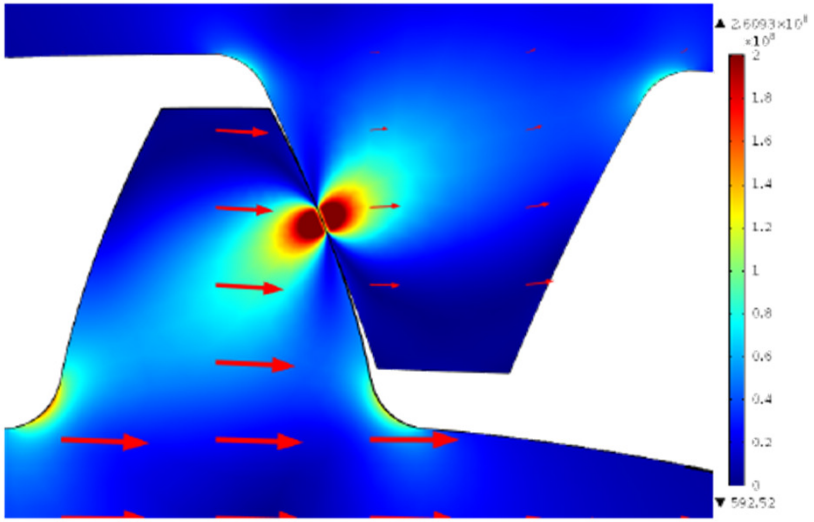
2D gear constraint equation



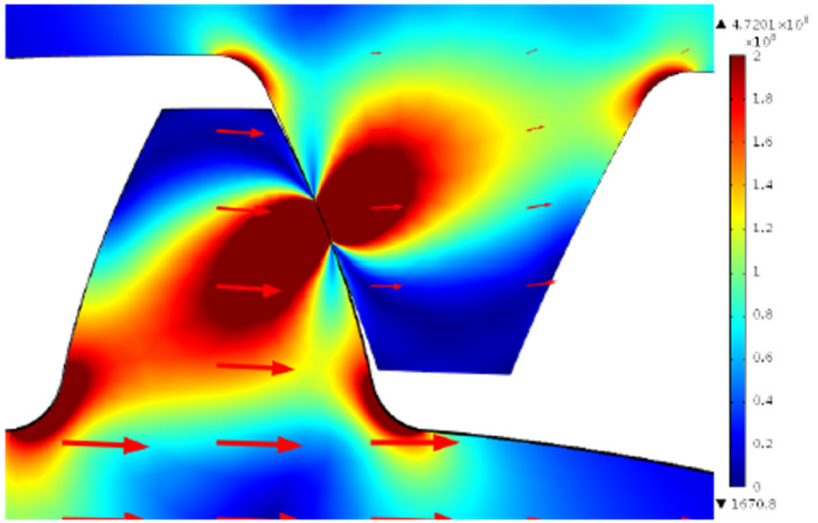
$$\ddot{\Phi} : (A_{70} \cdot \dot{v}_r)^T (\dot{r}_1 + r_{p1}\omega_1 \hat{v}_r) + (A_{70} \cdot v_r)^T \left(\ddot{r}_1 + r_{p1}\dot{\omega}_1 \hat{v}_r + r_{p1}\omega_1 \dot{\hat{v}}_r \right)$$

$$-(A_{70} \cdot \dot{v}_r)^T (\dot{r}_2 - r_{p2}\omega_2 \hat{v}_r) - (A_{70} \cdot v_r)^T \left(\ddot{r}_2 - r_{p2}\dot{\omega}_2 \hat{v}_r - r_{p2}\omega_2 \dot{\hat{v}}_r \right) = 0$$

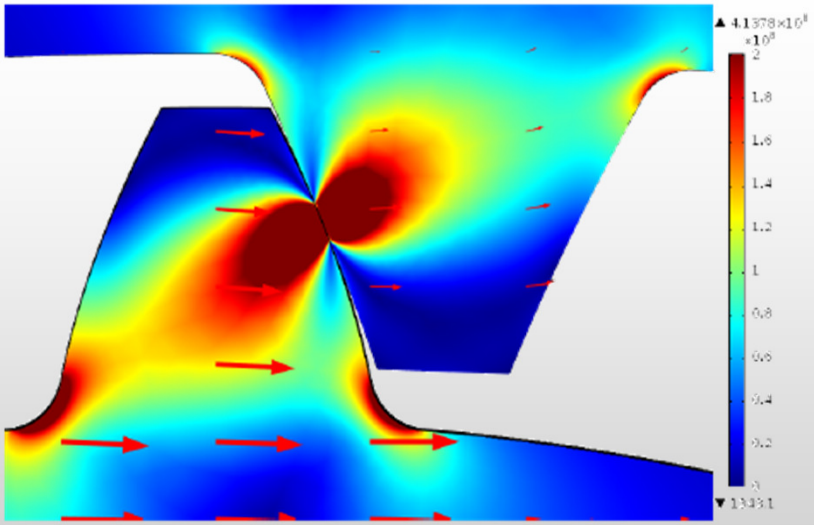
5. Results



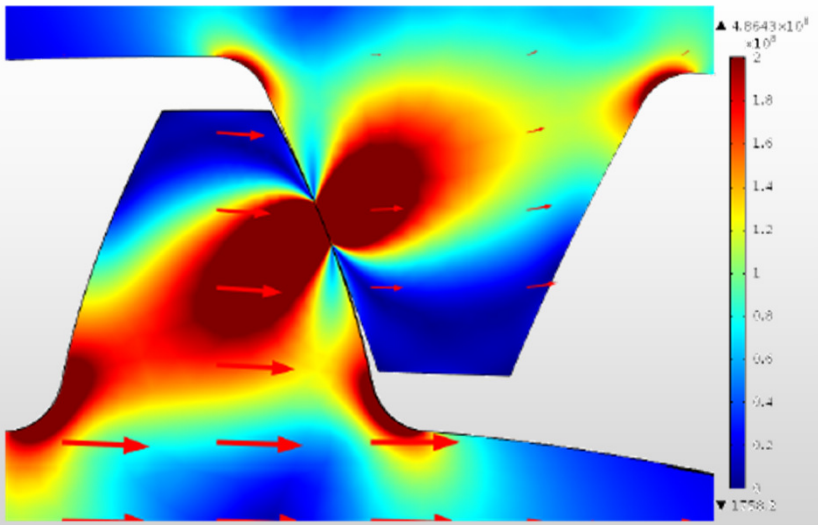
(b) 8 m/s



(f) 16 m/s



(d) 12 m/s



(h) 20 m/s